16–18 Work Backward Reinforce the meaning of points on the graph of the hull speed function (length at waterline, hull speed). Students should locate the point where $x = 24$ on the graph to find the speed of a boat with a 24-foot length at the waterline. Students should locate the point where $y = 6$ on the graph to find the length at the waterline of a boat with a hull speed of 6 knots.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to interpreting the graph of a square root function.

Answers

19. The hull speed is about 4 knots.
20. The points on the graph are not (1, 1), (4, 2), (9, 3), and so on. This is because the graph has been vertically stretched, which means the equation must include a transformation of some type.

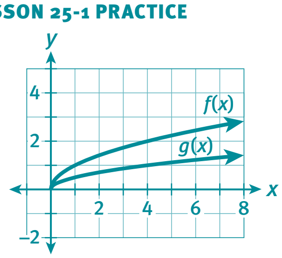
ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 25-1 PRACTICE

21.



ADAPT

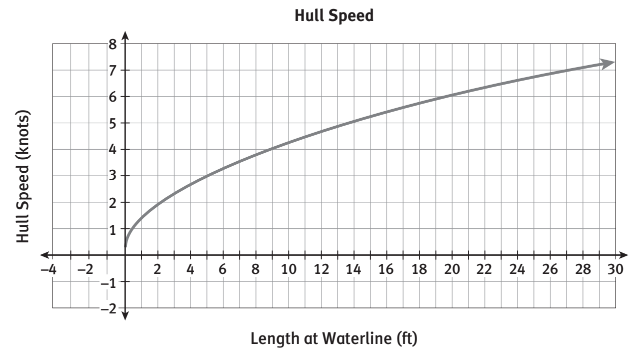
Check students' answers to the Lesson Practice to ensure that they understand the graphs of square root functions and their transformations. Additionally, students should understand the connection between the domain and range of the parent function and the domain and range of the transformations of the parent function. As additional practice, provide students with the domain and range of a square root function and have students write a square root function that would match.

ACTIVITY 25 *continued*

My Notes

Lesson 25-1
Square Root Functions

The graph of the hull speed of a boat H is shown below. You also sketched this graph in Item 1a.



16. Use the graph to estimate the hull speed of a boat that is 24 feet long at the waterline.
The hull speed is about 6.5 knots.

17. Use the graph to estimate the length at the waterline of a boat whose hull speed is 6 knots.
The boat would be about 20 feet long at the waterline.

18. Write an equation that could be solved to determine the length at the waterline of a boat with a hull speed of 6 knots.
$1.34\sqrt{x} = 6$

Check Your Understanding

19. Use the graph at the top of the page to estimate the hull speed of a boat that is 9 feet long at the waterline.

20. Explain how you can tell from the graph above that the equation relating the hull speed and the length of the boat is not $H(x) = \sqrt{x}$.

LESSON 25-1 PRACTICE

21. Graph $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{2}\sqrt{x}$ on the same axes.

22. Describe $g(x)$ as a transformation of $f(x)$. What are the domain and range of each function?

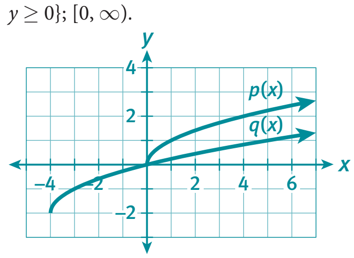
23. Graph $p(x) = \sqrt{x}$ and $q(x) = \sqrt{x+4} - 2$ on the same axes.

24. Describe $q(x)$ as a transformation of $p(x)$. What are the domain and range of each function?

25. **Reason abstractly.** Write a square root function that has a domain of $x \geq 7$ and a range of $y \geq 2$. Use a graphing calculator to confirm that your function meets the given requirements.

22. The graph of g is a vertical shrink of f by a factor of $\frac{1}{2}$. Both graphs contain the point (0, 0). Domain of both functions: $x \geq 0$; $\{x \mid x \in \Re, x \geq 0\}$; $[0, \infty)$; range of both functions: $y \geq 0$; $\{y \mid y \in \Re, y \geq 0\}$; $[0, \infty)$.

23.



24. The graph of q is a translation of p, 4 units to the left and 2 units down. Domain of p: $x \geq 0$; $\{x \mid x \in \Re, x \geq 0\}$; $[0, \infty)$; range of p: $y \geq 0$; $\{y \mid y \in \Re, y \geq 0\}$; $[0, \infty)$; domain of q: $x \geq -4$; $\{x \mid x \in \Re, x \geq -4\}$; $[-4, \infty)$; range of q: $y \geq -2$; $\{y \mid y \in \Re, y \geq -2\}$; $[-2, \infty)$; domain and range of q: $x \geq -4$ and $y \geq -2$.

25. $f(x) = a\sqrt{x-7} + 2$ for $a > 0$. Answer is acceptable without the a.

My Notes

Learning Targets:

- Solve square root equations.
- Identify extraneous solutions.

SUGGESTED LEARNING STRATEGIES: Note Taking, Identify a Subtask, Marking the Text, Predict and Confirm, Create Representations

To solve *square root equations*, follow these steps.

Step 1: Isolate the radical term.
Step 2: Square both sides of the equation.
Step 3: Solve for the unknown(s).
Step 4: Check for extraneous solutions.

MATH TIP

An *extraneous solution* can be introduced when you square both sides of an equation to eliminate the square root. The resulting equation may not be equivalent to the original for all values of the variable.

Example A

Solve the equation $\sqrt{x-3}+4=9$.

$$\sqrt{x-3}+4=9$$

Step 1: Isolate the radical. $\quad \sqrt{x-3}=5$

Step 2: Square both sides. $\quad (\sqrt{x-3})^2=(5)^2$

Step 3: Solve the equation. $\quad x-3=25$, so $x=28$

Step 4: Check the solution. $\quad \sqrt{28-3}+4\overset{?}{=}9$
$\quad 5+4=9$

Example B

Solve the equation $x=(x+1)^{\frac{1}{2}}+5$.

Step 1: Isolate the radical. $\quad x=(x+1)^{\frac{1}{2}}+5$
$\quad x-5=(x+1)^{\frac{1}{2}}$

Step 2: Square both sides. $\quad (x-5)^2=\left[(x+1)^{\frac{1}{2}}\right]^2$
$\quad x^2-10x+25=x+1$

Step 3: Solve for x. $\quad x^2-11x+24=0$
$\quad (x-3)(x-8)=0$

possible solutions $\quad x=3, 8$

Step 4: Check the possible solutions. $\quad 3\overset{?}{=}\sqrt{3+1}+5$
$\quad 3\neq 2+5$
$\quad 8\overset{?}{=}\sqrt{8+1}+5$
$\quad 8=3+5$

Only $x=8$ is a solution; $x=3$ is an extraneous solution.

WRITING MATH

You can write $\sqrt{x}$ as "x to the $\frac{1}{2}$ power," namely, $x^{\frac{1}{2}}$.

MINI-LESSON: Square Root Equations with More Than One Radical

For students who want to extend their learning by exploring more difficult square root equations, a mini-lesson is available.

See the Teacher Resources at SpringBoard Digital for a student page for this mini-lesson.

Lesson 25-2

PLAN

Pacing: 1 class period
Chunking the Lesson
Example A
Example B
#1–2
Check Your Understanding
#6–7
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Have students solve each of the following equations by taking the square root of each side. Emphasize the algebraic steps that are taken prior to taking the square root so that the quadratic term or binomial is isolated.

1. $4x^2-9=0 \quad \left[x=\pm\frac{3}{2}\right]$
2. $(x+2)^2+3=12 \quad [x=5 \text{ or } x=-1]$
3. $\frac{1}{3}(x-4)^2=-9 \quad [\text{no solution}]$

Example A Note Taking Make sure that students do not make the mistake of thinking $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$. They may confuse addition of radicals with multiplication of radicals. In multiplication, it is true that $\sqrt{ab}=\sqrt{a}\cdot\sqrt{b}$ when a and $b>0$. However, the same assumption is not true for addition of radicals.

Example B Note Taking, Create a Plan, Visualization. Many students continue to simplify an expression such as $(x-5)^2$ incorrectly even when reminded of this common error. An alternative to squaring both sides using exponent notation is to actually multiply each side by itself:
$(x-5)(x-5)=\sqrt{x+1}\cdot\sqrt{x+1}$

Universal Access

Have students study these examples and then hold a whole-class discussion, asking questions such as the following:

- Why do you square both sides of the equation to solve a square root equation?
- Why do you isolate the radical term? What does the word *isolate* mean?
- How does the order of operations help you identify which steps to do first when solving a radical equation?

As appropriate, relate these questions to the solutions of quadratic equations by taking the square root of both sides.

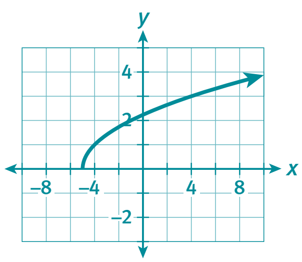
1–2 Marking the Text, Predict and Confirm, Create Representations, Quickwrite, Debriefing This brief application problem will assess students' ability to read the problem closely. Be sure that students use 24 ft, the length at the waterline, in their equation, not 27 ft. This item also provides an opportunity to assess communication skills. Some students may also wish to solve the equation graphically.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to solving square root equations and the connection between algebraic and graphical solutions.

Answers

3. a. $x = 17$
   b. $x = -1$
4. $x = -5$; The solution to the equation is the x-intercept of the graph.



5. Since the equation has no solution, the y-value will never equal 0. This means the graph will never intersect the x-axis.

My Notes

Lesson 25-2
Solving Square Root Equations

Try These A–B

Solve each equation.

a. $2 - \sqrt{x+1} = -5$
$x = 48$

b. $\sqrt{x+4} = x - 8$
$x = 12$; $x = 5$ is extraneous.

c. $(x+6)^{\frac{1}{2}} = -x$
$x = -2$; $x = 3$ is an extraneous solution.

d. $(x+4)^{\frac{1}{2}} + 1 = 0$
$x = -3$ is an extraneous solution, so the equation has no solution.

1. Solve the hull speed equation you wrote in Item 18 of the previous lesson.
$x = 20.05$ feet

2. Construct viable arguments. Maggie claims that her sailboat *My Hero* has a hull speed of 7 knots. The length of her boat at the waterline is 24 feet. Is Maggie's claim reasonable? Explain why or why not.
Solve the equation $1.34\sqrt{x} = 7$. The solution is $x = 27.3$ feet. The length at the waterline of Maggie's boat is only 24 feet, so this is not a reasonable claim.

Check Your Understanding

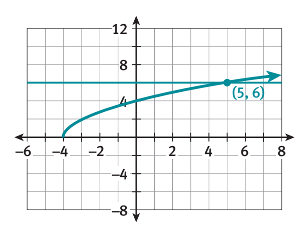
3. Solve each equation.
a. $(x-1)^{\frac{1}{2}} = 4$ **b.** $x + \sqrt{2x+3} = 0$

4. Solve the equation $0 = \sqrt{x+5}$, and then use transformations to sketch the graph of $f(x) = \sqrt{x+5}$. Make a connection between graphing $y = \sqrt{x+5}$ and solving the equation $0 = \sqrt{x+5}$.

5. Use your solution to the equation in Try These A–B part d to predict where the graph of $f(x) = (x+4)^{\frac{1}{2}} + 1$ will intersect the x-axis. Explain your reasoning.

You can also use technology to help you solve equations.

6. Solve the equation $2\sqrt{x+4} = 6$ using a graphing calculator. Enter the left side as one function and the right side as another function. Label the point where the graphs intersect.

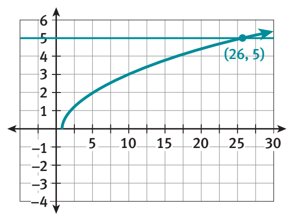


$x = 5$

Lesson 25-2
Solving Square Root Equations

My Notes

7. Solve the equation $\sqrt{x-1}=5$ using the same method as in Item 6.



$x = 26$

Check Your Understanding

8. Solve the equation $2\sqrt{x+1}=4$ using a graphing calculator. Include a sketch of the graph to support your answer.

9. In Example B, one of the two possible solutions found to the equation was an extraneous solution. Use a graphing calculator to solve the equation in Example B. Does the graph support the algebraic conclusion? Explain your reasoning.

10. In Item 6, you entered the left side of the equation as one function and the right side as another function and then found the point of intersection. Why is the x-coordinate of the point of intersection the solution to the equation?

LESSON 25-2 PRACTICE

Solve each equation algebraically. Identify any extraneous solutions. Check your solutions using a graphing calculator.

11. $(x-6)^{\frac{1}{2}}=4$

12. $3\sqrt{x+2}-7=5$

13. $x+\sqrt{x+3}=3$

14. $\sqrt{x-2}+7=4$

15. Explain how graphing $f(x)=\sqrt{x-2}+7$ and $g(x)=4$ supports your algebraic solution to the equation in Item 14.

16. **Make sense of problems.** The approximate intersection of the graphs of $f(x)=\sqrt{x+7}$ and $g(x)=x-1$ is (4.4, 3.4). Therefore, $x=4.4$ is the approximate solution to what equation?

LESSON 25-2 PRACTICE

11. $x=22$
12. $x=14$
13. $x=1$; $x=6$ is extraneous
14. no solution; $x=11$ is extraneous
15. The graphs do not intersect, which supports no solution.
16. $\sqrt{x+7}=x-1$

ACTIVITY 25 Continued

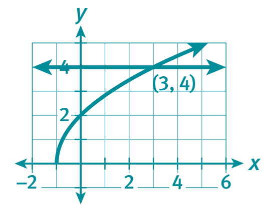
6–7 Create Representations, Simplify the Problem Point out that first changing the equation $2\sqrt{x+4}=6$ to $\sqrt{x+4}=3$ will result in the same graphical solution since the two equations are equivalent. Likewise, $\sqrt{x-1}=5$ and $x-1=25$ will result in the same graphical solution. Doing this will emphasize to students that the process of solving an equation algebraically means creating a series of equivalent equations until one side of the equation is equal to the variable itself.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to solving a square root equation graphically.

Answers

8. $x=3$;



9. Yes; the graph supports the solution because there is only one point of intersection. The x-value of that point is 8, which was the only valid solution found.

10. The intersection point is the only place where the graphs are equal, and therefore where the functions are equal. Since you are solving for x, the x-value of the intersection point is the solution to the equation.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand concepts related to solving square root equations both algebraically and graphically. Have students create a graphic organizer that summarizes the steps for using both solution methods to solve square root equations.

Lesson 25-3

PLAN

Pacing: 1 class period

Chunking the Lesson

#1 #2–4

Check Your Understanding

#8–9

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Have students explain how the graphs of the following transformations relate to the parent function $f(x) = x^3$.

1. $f(x) = \frac{1}{3}x^3$
   [*vertical compression by* $\frac{1}{3}$]
2. $f(x) = (x - 1)^3 + 4$
   [*shift 1 unit right and 4 units up*]
3. $f(x) = -2x^3$
   [*reflection across the x-axis and vertical stretch by a factor of 2*]

1 Activating Prior Knowledge, Create Representations, Quickwrite

Emphasize plotting key points for the parent cube root function, such as (0, 0), (1, 1), (8, 2), and (27, 3), when sketching graphs.

Emphasize that while this is a new parent function, the effect of these transformations on the parent function $f(x) = x^3$ is not a new concept.

My Notes

MATH TIP

The function for the radius used here is the inverse of the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$, with V represented by x.

MATH TIP

$\sqrt[3]{x}$ can be written as "x to the $\frac{1}{3}$ power," or $x^{\frac{1}{3}}$.

Learning Targets:

- Graph transformations of the cube root function $y = \sqrt[3]{x}$.
- Identify key features of a graph that models a relationship between two quantities.

SUGGESTED LEARNING STRATEGIES: Create Representations, Note Taking, Look for a Pattern

The function $r(x) = \sqrt[3]{\frac{3}{4\pi}x}$ represents the length of the radius of a sphere as a function of its volume, represented here by x. An approximation of this function is $r(x) = \sqrt[3]{0.24x}$.

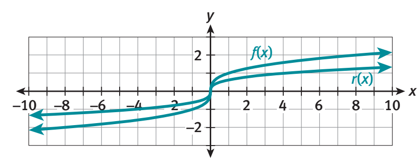
1. The radius function is a transformation of the parent cube root function $f(x) = \sqrt[3]{x}$.

a. Write $r(x)$ as a product in the form $r(x) = a\sqrt[3]{x}$ or $r(x) = ax^{\frac{1}{3}}$ so that it is easier to see the relationship between it and the parent function. Round a to one decimal place.

$r(x) = 0.6\sqrt[3]{x}$ or $r(x) = 0.6x^{\frac{1}{3}}$

b. Graph $r(x)$ and $f(x)$ on the same axes. How do these graphs compare to each other? How would the graphs of $h(x) = x^3$ and $j(x) = 0.6x^3$ compare?

$r(x)$ is a vertical shrinking of $f(x)$. Both graphs contain the point (0, 0). $j(x)$ is similar to $r(x)$ and is a horizontal stretch of $h(x)$. Both h and j contain (0, 0).



c. What are the domain and the range of $f(x)$? Of $r(x)$? Write your answers as inequalities, in set notation, and in interval notation.

domain of f: all real numbers; $\{x \mid x \in \Re\}$; $(-\infty, \infty)$

range of f: all real numbers; $\{x \mid x \in \Re\}$; $(-\infty, \infty)$

The domain and range of r are the same as f.

d. Describe the transformations of $h(x) = x^3$ that result in the following functions.

i. $j(x) = (-2x)^3$
reflection across the x-axis and vertical stretch by a factor of 8

ii. $k(x) = (x + 5)^3$ horizontal translation 5 units to the left

iii. $m(x) = x^3 - 4$ vertical translation down 4 units

iv. $n(x) = 2(0.5x + 4)^3 - 6$
vertical shrink by a factor of $\frac{1}{4}$, horizontal translation 4 units to the left, vertical translation 6 units down

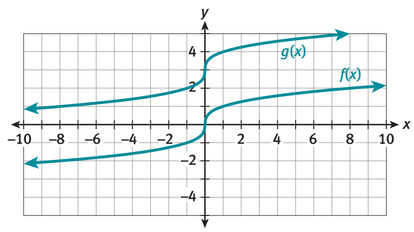
Lesson 25-3
Cube Root Functions

ACTIVITY 25
continued

My Notes

2. How does the graph of $g(x) = \sqrt[3]{x} + 3$ compare to the graph of $f(x) = \sqrt[3]{x}$? How would the graphs of $h(x) = x^3$ and $j(x) = x^3 + 3$ compare?
g is a vertical translation of the parent function 3 units up. Similarly, j is a vertical translation of h 3 units up.

3. Sketch $g(x)$ and $f(x)$ from Item 2 on the same axes below.



4. What are the domain and range of each function?
The domain and range of each function are all real numbers.

Check Your Understanding

5. Why is the domain of a cube root function all real numbers, while the domain of a square root function is restricted to only nonnegative numbers?

6. **Attend to precision.** In Item 1, x represents the volume of a sphere and $r(x)$ the volume. Given this context, should the domain and range of $r(x)$ be restricted? Explain your reasoning.

7. Write an equation that could be solved to find the volume of a sphere that has a radius of 2 inches. Write your equation two ways using a rational exponent and a radical.

8. Describe the transformations of $f(x) = \sqrt[3]{x}$ and $h(x) = x^3$ that result in the functions listed below.

a. $g(x) = -\sqrt[3]{x-3}$ and $j(x) = -(x-3)^3$
The graphs of $f(x)$ and $h(x)$ have been reflected across the x-axis and translated 3 units to the right.

b. $p(x) = 4\sqrt[3]{x} - 1$ and $q(x) = 0.25(4x)^3 - 1$
The graph of $f(x)$ has been vertically stretched by a factor of 4 and shifted down 1 unit. The graph of $h(x)$ has been vertically stretched by a factor of 16 and shifted down 1 unit.

9. Without graphing, state the domain and range of the functions in Item 8.
The domain and range of each function are all real numbers.

ACTIVITY 25 Continued

2–4 Visualization, Create Representations Some students view the end behavior of the cube root function as being asymptotic since it increases or decreases so slowly. If necessary, create a table of values to help students understand that the function will both decrease and increase without bound as x increases and decreases without bound.

Check Your Understanding
Debrief students' answers to these items to ensure that they understand concepts related to the domain and range of square root functions and the transformations of the graph of the parent cube root function.

Answers
5. You can take the cube root of a negative number, but you cannot take the square root of a negative number.

6. Yes; the radius and volume must be greater than or equal to 0, so the domain and the range are both $[0, \infty)$.

7. $2 = \left(\frac{3x}{4\pi}\right)^{\frac{1}{3}} \simeq 2 = (0.24x)^{\frac{1}{3}}$

$2 = \sqrt[3]{\frac{3x}{4\pi}} \approx 2 = \sqrt[3]{0.24x}$

so $x = \frac{32}{3}\pi$

8–9 Note Taking, Visualization, Debriefing Make sure that students are using proper vocabulary when describing transformations of graphs. Since the parent cube root function and the parent cubic function are both odd functions, this is a good opportunity to review the concept of odd functions. Odd functions exhibit symmetry about the origin and have the property that $f(-x) = -f(x)$ for all domain values of the function.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to the domain and range of a cube root function and are able to interpret cube root functions in a real-world context.

Answers

10. $f(x)$: vertical stretch by a factor of 2, translation 4 units to the left and 7 units down

$h(x)$: $2(x+4)^{3\,-7}$ vertical shrink by a factor of 2, translation 4 units left and 7 units down

11. Answers are acceptable without a, but must have a negative sign in front of the radical.

a. $f(x) = -a\sqrt[3]{x-5}$ for $a > 0$.
b. $g(x) = -a(x-5)^3$

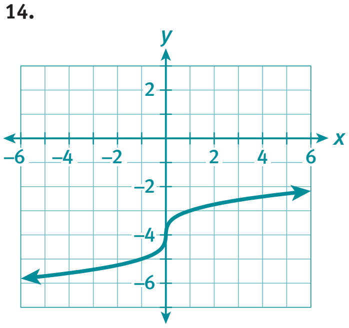
ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 25-3 PRACTICE

12. about 1.1 units
13. $y = -\sqrt[3]{x-2} + 1$
14.



domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand concepts related to graphing transformations of cube root functions. Additionally, students should be able to identify the domain and range of these functions. Have students compare and contrast the graphs of cube root and square root functions, their transformations, and their domains and ranges.

My Notes

Check Your Understanding

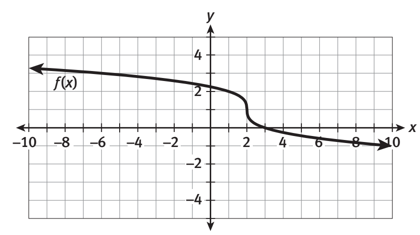
10. Describe the transformations of $f(x) = \sqrt[3]{x}$ that result in the function $f(x) = 2\sqrt[3]{x+4} - 7$. Also, describe the transformations of $h(x) = x^3$ that result in the function $h(x) = 2(x+4)^3 - 7$.

11. a. Write an equation for a cube root function that has been reflected across the x-axis and shifted horizontally 5 units to the right.
b. Write an equation for a cubic function that has been reflected across the x-axis and shifted horizontally 5 units to the right.

LESSON 25-3 PRACTICE

12. Use your graph in Item 1b to estimate the radius of a sphere that has a volume of 6 cubic units.

13. Assuming the graph below has not undergone a stretch or a shrink, write a possible equation for the function shown.



14. Sketch the graph of $h(x) = \sqrt[3]{x} - 4$. Then state the domain and range.

15. Sketch the graph of $p(x) = \sqrt[3]{x+5} + 2$. Then state the domain and range.

16. Consider the statement below.

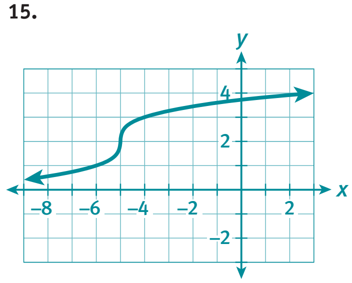
If a cube root function is reflected across the x-axis and then across the y-axis, the resulting graph is the same as the graph of the original cube root function before the transformations.

Do you agree with the statement? If not, explain why. If you agree, write an algebraic expression that represents the relationship described in the statement.

17. Make use of structure. If you solve the equation $\sqrt[3]{x} = 0$, you find that the graph of $f(x) = \sqrt[3]{x}$ intersects the x-axis at $x = 0$. Where does the graph of $f(x) = \sqrt[3]{x+6}$ intersect the x-axis? Explain your reasoning.

18. Describe the translations of $f(x) = x^3$ that result in the following functions.
a. $f(x) = (-3x)^3$
b. $f(x) = \left(\frac{1}{3}x\right)^3$
c. $f(x) = 4\left(-\frac{1}{2x}\right)^3 - 5$

15.



domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

16. Yes; $-\sqrt[3]{-x} = \sqrt[3]{x}$

17. The graph intersects the x-axis at $x = -6$ because the graph has been shifted to the left 6 units. You can also find the x-intercept by solving the equation $0 = \sqrt[3]{x+6}$.

18. a. reflection across the x-axis, vertical stretch by a factor of 27
b. vertical shrink by a factor of $\frac{1}{27}$
c. reflection across the x-axis, vertical shrink by a factor of $\frac{1}{8}$, vertical shift down 5 units

My Notes

Learning Targets:

- Solve cube root equations.
- Check the reasonableness of solutions.

SUGGESTED LEARNING STRATEGIES: Note Taking, Identify a Subtask, Marking the Text, Create Representations

The steps for solving *cube root equations* are very similar to those for solving square root equations.

Step 1: Isolate the radical term.

Step 2: Cube both sides of the equation.

Step 3: Solve for the unknown(s).

Example A

Solve the equation $4\sqrt[3]{2x-1}-5=3$.

$$4\sqrt[3]{2x-1}-5=3$$

Step 1: Isolate the radical.

$$4\sqrt[3]{2x-1}=8$$

Add 5 to both sides, and then divide both sides by 4.

$$\sqrt[3]{2x-1}=2$$

Step 2: Cube both sides.

$$\left(\sqrt[3]{2x-1}\right)^3=(2)^3$$

Step 3: Solve the equation.

$$2x-1=8$$
$$2x=9$$
$$x=\frac{9}{2}$$

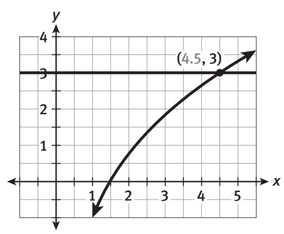
Example B

Use a graphing calculator to solve the equation in Example A.

Step 1: Enter the left side of the equation as one function and the right side as a second function.

Step 2: Find the point of intersection.

Step 3: The x-coordinate of the point of intersection is the solution.



Solution: $x=4.5$

ACTIVITY 25 Continued

Lesson 25-4

PLAN

Pacing: 1 class period

Chunking the Lesson

Example A

Example B

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Have students solve each of the following equations by taking the cube root of each side. Emphasize the algebraic steps that are taken prior to taking the cube root so that the cubic term or binomial is isolated.

1. $8x^3=-27$ $\left[x=-\frac{3}{2}\right]$
2. $-\frac{1}{2}(x-2)^3+4=36$ $[x=-2]$
3. $3(x-4)^3-4=-1$ $[x=5]$

Example A Note Taking, Predict and Confirm Work through this example with students, being sure to note that the solution process is similar to equations involving square roots. Ask students to consider whether there may potentially be differences in the solution processes when working with different examples.

Example B Debriefing, Predict and Confirm, Visualization Prior to graphing the left and right sides of the equation, ask students to consider the maximum and minimum intersection points between a cube root and a constant function. Emphasize that there must be exactly one intersection point since the range of a cube root function is all real numbers and the range of a constant function is only one number.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to solving cube root equations.

Answers

1. **a.** $x = 126$
   b. $x = 3$
2. The solution to the equation is $x = 512$.
3. Sample answer: The calculator is more efficient in this case because the algebraic solution involves squaring a binomial, combining like terms, and then factoring the resulting trinomial.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning. See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 25-4 PRACTICE

4. $x = 8$
5. $x = -4$
6. $x = 32$
7. No. The viewing window is too small to see the point of intersection. If you expand the window, the point of intersection will reveal an x-value of 126, which corresponds to the solution in Item 1a.
8. Solve the equation $2 = \sqrt[3]{0.24x}$. The solution is $x = 33.3$ cubic inches; so the volume is only 33.3 cubic inches, Marcus's claim is not reasonable.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand concepts related to solving cube root equations both algebraically and graphically. As additional practice, create a matching activity in which students match the graphical solution of a cube root equation with the cube root equation. Do not allow students to use a graphing calculator when making the matches.

My Notes

Lesson 25-4
Solving Cube Root Equations

Try These A–B

Solve each equation. Use algebraic techniques or a graphing calculator.

a. $4 + (x-1)^{\frac{1}{3}} = 2$
$x = -7$

b. $7\sqrt[3]{2x+5} = 21$
$x = 11$

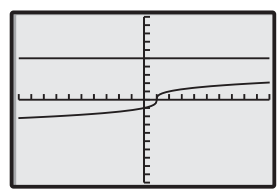
Check Your Understanding

1. Solve each equation.
   a. $\sqrt[3]{x-1} = 5$ **b.** $\sqrt[3]{3x-1} - 6 = -4$
2. **Reason quantitatively.** Kari graphed the functions $y_1 = \sqrt[3]{x} - 4$ and $y_2 = \frac{\sqrt[3]{x}}{2}$ on her graphing calculator and found that the intersection of the graphs is (512, 4). What does this tell you about the solution to the equation $\sqrt[3]{x} - 4 = \frac{\sqrt[3]{x}}{2}$?
3. In Item 2, Kari used a graphing calculator to solve the problem. In this particular case, is the use of a graphing calculator more efficient than the use of algebraic techniques? Explain your reasoning.

LESSON 25-4 PRACTICE

For Items 4–6, solve each equation algebraically. Check your solutions using a graphing calculator.

4. $\sqrt[3]{x} + 3 = 5$
5. $2 + \sqrt[3]{x+5} = 3$
6. $3\sqrt[3]{2x} = 12$
7. Solving the equation $\sqrt[3]{x-1} = 5$ using a graphing calculator set to a standard 10-by-10 viewing window yields the following graph.



   Does this graph contradict your solution to Item 1a? Explain your reasoning.
8. **Critique the reasoning of others.** Marcus claims that a softball with a radius of approximately 2 inches has a volume of 40 cubic inches. Is Marcus's claim reasonable? Solve the equation you wrote in Item 7 of the previous lesson to support your answer.

ACTIVITY 25 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 25-1

1. The function $L(x) = \sqrt{\frac{x}{6}}$ represents the length of a side of a cube whose surface area is x square units.

a. Write $L(x)$ as a transformation of the parent square root function $f(x) = \sqrt{x}$.

b. Graph $L(x)$ and $f(x)$ on the same axes. How do these graphs compare to each other?

c. The x-intercept of $f(x)$ is $x = 0$. What is the x-intercept of $L(x)$?

d. What is the domain of $L(x)$? Is the domain reasonable for this scenario?

2. Which of the following square root functions has a domain of $[3, \infty)$?

A. $f(x) = \sqrt{x} + 3$

B. $f(x) = \sqrt{x} - 3$

C. $f(x) = \sqrt{x+3}$

D. $f(x) = \sqrt{x-3}$

3. Explain why the range of $g(x) = -\sqrt{x}$ is $(-\infty, 0]$ rather than $[0, \infty)$. Draw a sketch to support your answer.

Lesson 25-2

4. a. Solve the equation $4 = \sqrt{\frac{x}{6}}$ to find the surface area of a cube whose sides are 4 cm long. Show your work.

b. Use the formula for finding the surface area of a cube, $S = 6s^2$, where s is the length of the side of the cube, to check your results from part a. Is your answer reasonable?

5. Solve each equation algebraically. Identify any extraneous roots.

a. $\sqrt{x-2} + 5 = 8$

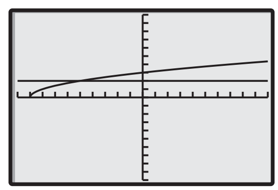
b. $\sqrt{x+2} + x = 0$

c. $x = \sqrt{3x-12} + 4$

d. $\sqrt{x+5} + 2 = 0$

6. Let $f(x) = \sqrt{x+5} + 2$. Sketch the graph of $f(x)$ using what you know about transformations. Identify the x-intercepts, if any, and tell whether your graph supports your answer to Item 5d.

7. The screen shot below shows the solution to a square root equation.



Assuming that the radical term is of the form $\sqrt{x \pm a}$ and that the point of intersection is $(-5, 2)$, write an equation that the graph could be used to solve.

ACTIVITY 25 Continued

ACTIVITY PRACTICE

1. a. $L(x) = 0.4\sqrt{x}$

b. The graph of L is a vertical shrinking of f. All the y-values have been multiplied by 0.4.

c. $x = 0$

d. $[0, \infty)$; Yes; since the surface area must be a non-negative number.

2. C

3. The y-values approach negative infinity because the graph has been reflected across the x-axis. Check students' sketches.

4. a. 96 cm^2

b. Yes; $S = 6(4)^2 = 96\ \text{cm}^2$

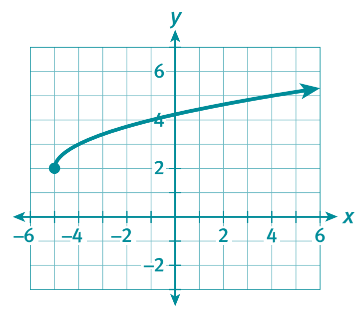
5. a. $x = 11$

b. $x = -1$; $x = 2$ is extraneous

c. $x = 4$ and $x = 7$

d. $x = -1$ is extraneous; The equation has no solution.

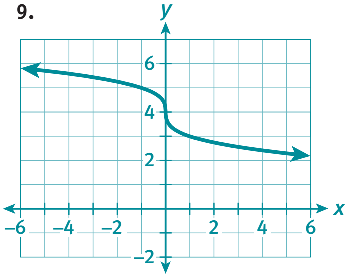
6. There are no x-intercepts, which supports no solution.



7. $2 = \sqrt{x+9}$

8. A

9.



10. The domain and range are both $(-\infty, \infty)$. This is true for all cube root functions since you can take the cube root of any real number.

11. It is not possible, because the domain for every cube root function is $(-\infty, \infty)$.

12. $y = \sqrt[3]{x+2} + 4$

13. **a.** $x = 27$

b. $x = 7$

14. $x = 25$

15. Sample answer: Problem 2; because the algebraic solution involves squaring a binomial, combining like terms, and then factoring the resulting trinomial; $x \approx 7.8$.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

Lesson 25-3

8. Which equation represents the following transformation?

the parent cube root function $f(x) = \sqrt[3]{x}$ *vertically stretched by a factor of 2*

A. $g(x) = \sqrt[3]{8x}$

B. $h(x) = \sqrt[3]{2x}$

C. $p(x) = \sqrt[3]{x-2}$

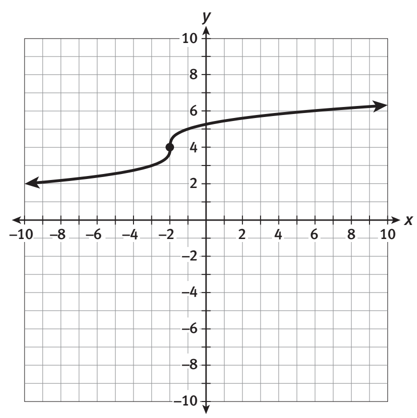
D. $q(x) = \sqrt[3]{x} + 2$

9. Sketch the graph of $f(x) = -\sqrt[3]{x} + 4$ using what you know about transformations.

10. Determine the domain and range of the function $f(x) = a\sqrt[3]{bx - c} + d$. Justify your answer.

11. If possible, give an example of a transformation that changes the domain of a cube root function. If not possible, explain why not.

12. Assuming the graph below represents a cube root function that has not been stretched or shrunk, write a possible equation for the function.



Lesson 25-4

13. Solve each equation using algebraic techniques. Show your work.

a. $\sqrt[3]{x} + 5 = 8$

b. $5\sqrt[3]{x} + 1 = 10$

14. To enter a cube root function into a graphing calculator, you can write the radical using an exponent of $\frac{1}{3}$. Use this fact to solve the equation $\sqrt[3]{x+2} + 9 = 12$.

MATHEMATICAL PRACTICES
Use Appropriate Tools Strategically

15. You are given the option on a math quiz to solve only one problem using a graphing calculator. You must solve the other problems using algebraic techniques. Which of the following would you choose to solve with the graphing calculator, and why? Be specific. Then solve the equation using a calculator.

Problem 1: $\sqrt{x} + 5 = 17$

Problem 2: $\sqrt{x} + 5 = x$

Problem 3: $\sqrt{x} + x = 5 + x$

Inverses: Roots, Squares, and Cubes

ACTIVITY 26

Swing, Swing, Swing

Lesson 26-1 Square Root Functions and Regressions

My Notes

Learning Targets:

- Graph and write the inverse of square root functions.
- Find a square root model for a given table of data.

SUGGESTED LEARNING STRATEGIES: Create Representations, Note Taking, Think-Pair-Share

You have studied linear functions and their inverses as well as logarithmic and exponential functions that are inverse functions. Let's review what we know about inverse functions f and g:

$$f(g(x)) = x \text{ for all } x \text{ in the domain of } g, \text{ and}$$
$$g(f(x)) = x \text{ for all } x \text{ in the domain of } f.$$

Now consider one of the radical functions you've just studied: square root functions.

Example A

Graph the inverse of $f(x) = x^{\frac{1}{2}}$. Then give the domain and range of both the function and its inverse.

Step 1: List four points on the graph of f.

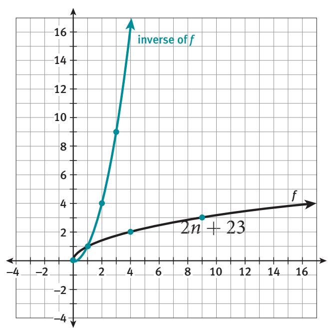
$(0, 0), (1, 1), (4, 2), (9, 3)$

Step 2: Interchange x and y for each point. These points will be on the graph of the inverse.

$(0, 0), (1, 1), (2, 4), (3, 9)$

Step 3: Connect the points.

Step 4: Consider the x- and y-values of the graphs to determine the domain and range of the functions.



Solution: The blue graph is the inverse of f. The domain of both the function and its inverse is $x \geq 0$. The range of both the function and its inverse is $y \geq 0$.

Example B

For $f(x) = x^{\frac{1}{2}}$, find f^{-1} algebraically. Then give the domain and range of f^{-1}.

Step 1: Let y represent $f(x)$. $\qquad y = x^{\frac{1}{2}}$

Step 2: Interchange x and y to form the inverse relationship. $\qquad x = y^{\frac{1}{2}}$

Step 3: Solve for y to find the inverse. $\qquad (x)^2 = \left(y^{\frac{1}{2}}\right)^2$

Assume the inverse is a function. $\qquad x^2 = y$

Solution: $f^{-1} = x^2$. The domain of f^{-1} is all real numbers and the range is $y \geq 0$.

MATH TIP

Recall that rational exponents are another way of writing radical expressions with

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$
$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

MATH TIP

Recall that if (x, y) is a point on the graph of f, then (y, x) is a point on the graph of its inverse.

MATH TIP

The typical notation for an inverse function is f^{-1}. It is extremely important to note that $f^{-1}(x) \neq \frac{1}{f(x)}$.

Common Core State Standards for Activity 26

HSA-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
HSA-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
HSF-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ★

ACTIVITY 26

Directed

Activity Standards Focus

In Activity 26, students investigate the inverse relationship between roots and powers. They graph and write the inverse of square root and quadratic functions. They graph and write the inverse of cube root and cubic functions. Throughout this activity, be sure students recall how the domain of a function is related to the range of the inverses of the function and vice versa.

Lesson 26-1

PLAN

Pacing: 1 class period

Chunking the Lesson

Example A
Example B
Check Your Understanding
#4–5 #6
Example C
#7–10
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Have students find the inverse of the following functions.

1. $f(x) = 14 - 2x$ $\qquad \left[f^{-1}(x) = -\frac{1}{2}x + 7\right]$

2. $f(x) = x + 5$ $\qquad [f^{-1}(x) = x - 5]$

3. $f(x) = x^2 + 4$ $\qquad \left[f^{-1}(x) = \sqrt{4 - x}\right]$

Example A Debriefing, Activating Prior Knowledge, Summarizing, Visualization This example demonstrates how to graph a radical function using a set of ordered pairs that satisfy its equation. Then it demonstrates how to graph the inverse of this radical function by interchanging, or switching, the x- and y-values for each of the ordered pairs. This example also describes the domain and range of a radical function and its inverse. You may want to spend some time discussing the relationships between the domain and range of this function and its inverses: *the domain of the function becomes the range of its inverse, and the range of the function becomes the domain of the inverse.*

Example B Debriefing, Summarizing

This example uses the same square root function as in Example A, yet it demonstrates how to solve for y algebraically in order to find the inverse function. The important thing to note here is that in this example, the domain of the inverse was found to be all real numbers, rather than just $x \geq 0$. Therefore, be sure to emphasize to students that if they are finding an inverse function algebraically, they need to restrict its domain to match the range of the original function.

ELL Support

The word *restrict* is important in these examples. You may want to explain to students whose primary language is not English that you are "limiting" the values of the domain of the inverse to match those of the range of the original function. If you do not restrict the domain of the inverse, then the inverse may not remain a function. Students will eventually explore this further.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to square root functions and their inverses.

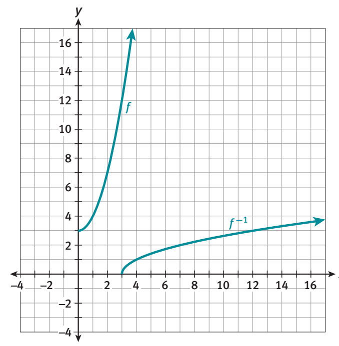
Answers

1. x^2; 0
2. $h^{-1} = \dfrac{2500x^2}{4489} \approx 0.5569x^2$ for $x \geq 0$.
3. domain of h and h^{-1}: $x \geq 0$; $\{x \mid x \in \Re, x \geq 0\}$; $[0, \infty)$
   range of h and h^{-1}: $y \geq 0$; $\{y \mid y \in \Re, x \geq 0\}$; $[0, \infty)$

My Notes

Try These A–B

a. Graph $f(x) = (x-3)^{\frac{1}{2}}$ using the values $x = 3$, 4, and 7. Then graph its inverse on the same axes.



b. For the function in part a, find f^{-1} algebraically.

$f^{-1} = x^2 + 3$

In Examples A and B, we found the inverse of $f(x) = x^{\frac{1}{2}}$ using two different techniques. In Example A, graphing the inverse resulted in a domain of $x \geq 0$. However, finding the inverse algebraically resulted in the function $f^{-1} = x^2$, which we already know has a domain of all real numbers. The explanation for this is subtle but still very important when defining the inverse of a function: The domain of the inverse function f^{-1} should be restricted to match the range of the original function.

Check Your Understanding

1. Complete this statement about the inverse of $f(x) = x^{\frac{1}{2}}$:
   $f^{-1}(x) =$ ____________ for $x \geq$ ____________.
2. Find the inverse of the function $h(x) = 1.34x^{\frac{1}{2}}$.
3. Give the domain and range for both h and h^{-1} in Item 2. Write your answers using inequalities, set notation, and interval notation.

Common Core State Standards for Activity 26 *(cont.)*

HSF-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
HSF-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
HSF-BF.B.4	Find inverse functions.
HSF-BF.B.4a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$.*
HSF-BF.B.4d (+)	Produce an invertible function from a non-invertible function by restricting the domain.

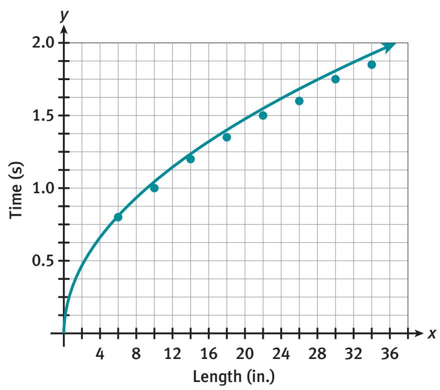
Lesson 26-1
Square Root Functions and Regressions

In a previous lesson, you learned how to use a graphing calculator to find a quadratic regression based on real world data. Let's do the same for a square root function.

A physics class conducted an experiment comparing the period of a pendulum to the length of the pendulum. The results of the experiment are given in the table below.

Length (in.)	6	10	14	18	22	26	30	34
Time (s)	0.8	1.0	1.2	1.35	1.5	1.6	1.75	1.85

4. Make a scatter plot of the data on the coordinate grid below.



5. What parent function does the path of the data points resemble?
$f(x) = \sqrt{x}$

Recall that a quadratic regression is the process of finding a quadratic function that best fits a set of data. We used a graphing calculator to perform quadratic regressions. A ***square root regression*** is a similar process and can also be performed using a graphing calculator.

6. Make use of structure. If a square root function of the form $f(x) = a\sqrt{x}$ is the best fit for the data graphed in Item 4, make a conjecture about the value of a. Explain your reasoning.
The graph has not been reflected, so $a > 0$. Also, based on the data points, the graph has been vertically shrunk so $a < 1$. So, $0 < a < 1$.

My Notes

CONNECT TO PHYSICS

The period of a pendulum is the length of time it takes to make one cycle, a complete swing back and forth. The period varies with the length of the pendulum, although other factors, which are not taken into account here, can affect the period.

MATH TERMS

A **square root regression** is the process of finding a square root function that best fits a set of data.

4–5 Think-Pair-Share, Debriefing, Activating Prior Knowledge For Item 4, have students work in pairs to make a scatter plot of the ordered pairs from the table onto the coordinate grid. Tell students to draw a "curve of best fit" on the scatter plot. If students need assistance with Item 5, take the time for a quick review of some of the parent functions. Have students share and compare their results with the class before moving on to Item 6.

6 Levels of Questions, Debriefing Ask students to sketch the parent function $f(x) = \sqrt{x}$ onto the coordinate grid, along with their scatter plot. In the parent function, the value of $a = 1$. This will help students make comparisons between the two graphs in order to make conjectures about the value of a. Students will need to remember what is meant by a reflection and transformation of a graph.

Some questions you could ask to guide the students in making their conjectures are the following: In what Quadrant does the graph lie, and what role does the sign of a play in that? What type of transformation has taken place between this function and the parent square root function, and what might that tell you about the value of a? Have a class discussion about the answers to these questions, as a means of making conjectures regarding a.

Developing Math Language

In the field of psychology, *regression* refers to a defense mechanism in which a person reverts back to a previous stage of development. However, in mathematics, *regression* involves analyzing the relationship between the dependent variable and one or more independent variables of a function. Oftentimes, when the graph of a function is compared to its parent function, predictions can be made regarding its regression. More specifically, in this lesson, students will perform a non-linear regression using their graphing calculators to determine the equation of the curve that is the best fit to the scatter plot.

Example C Chunking the Activity, Close Reading Have students follow the detailed list of steps to perform a power regression for this function on their graphing calculators. Students who are more comfortable with the technology can help other students who may be more intimidated by navigating through the various screens, menus, and submenus required. One thing to emphasize to students while performing the regression is to double-check their data entries for L1 and L2. If there is a mistake, they may get either an incorrect equation or even an error message.

7–10 Discussion Groups, Group Presentation, Debriefing Have students work in small groups for these items. For Item 7, suggest that students use the same x-values as shown in the table. This will enable them to see just how close of a "good fit" the graph is to the scatter plot points. For Item 8, do not expect students to state that the domain and range do not extend indefinitely to infinity, or that the upper limit cannot be determined. You could use this as a moment to briefly discuss what is meant by a limit, and how they are commonly used in calculus. For Item 9, students replace x in the function with 40 and solve for y. For Item 10, students replace y in the function with 3 and solve for x. Note: Item 10 states to round to the nearest half inch. Many students will probably have 82.6, and you may need to explain the answer is 82.5, due to rounding to the nearest half inch. After giving students a few minutes to respond to these items, have a class discussion using a variety of groups to present their responses.

Technology Tip

Items 9 and 10 can both be solved algebraically. However, remind students that a great way to check their answers is to use the TABLE feature of a graphing calculator to scroll down through the list of values that correspond with this function. For example, for Item 9, students could scroll down until $x = 40$; or they could use the TBLSET and have the calculator ASK for the x-value, type in 40, and hit ENTER, to get the corresponding y-value. For Item 10, students could scroll down until $y = 3$ to find the corresponding x-value.

For additional technology resources, visit SpringBoard Digital.

My Notes

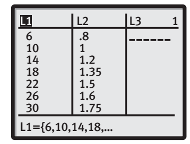
TECHNOLOGY TIP

PwrReg stands for *power regression*. It can be used for any regression of the form $y = ax^b$, when there are no translations.

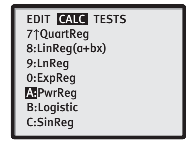
Example C

Use a graphing calculator to perform a regression for the pendulum data and determine the type of function that is the best fit.

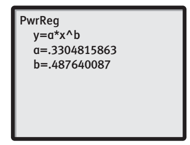
Step 1: Press STAT to open the statistics menu. Choose **Edit** by pressing ENTER. Enter the data: the length data as L1 and the time data as L2.



Step 2: Press STAT again. Move the cursor to highlight **Calc** and then scroll down the submenu to select **A:PwrReg**. Press ENTER.



Step 3: The calculator displays the values of a and b for the standard form of a power function that best fits the data. Use the values of a and b to write the equation. Round all values to two decimal places.



Solution: The equation of the regression is $y = 0.33x^{0.49} \approx 0.33\sqrt{x}$. A square root function is a good fit for the data.

7. Graph the square root model $y = 0.33\sqrt{x}$ on the coordinate grid in Item 4. Does your graph support a "good fit"? Explain your reasoning.
See students' graphs. Yes, the graph fits very closely to all of the data points in the scatter plot.

8. Give the domain and range for your square root regression. Write your answers using inequalities, set notation, and interval notation.
Domain: $x \geq 0$; $\{x \mid x \in \Re, x \geq 0\}$; $(0, \infty)$
Range: $y \geq 0$; $\{y \mid y \in \Re, y \geq 0\}$; $(0, \infty)$
However, the domain and range do not extend indefinitely to infinity, but the precise upper limit cannot be determined.

9. Use the square root model to predict the period of a pendulum that is 40 inches long. Round your answer to two decimal places.
2.09 seconds

10. Use the regression equation $y = 0.33\sqrt{x}$ to find the length of a pendulum that has a period of 3 seconds. Round your answer to the nearest half inch.
82.5 inches

My Notes

Check Your Understanding

11. The regression equation obtained from the calculator in Example C is $y = 0.33x^{0.49}$. Explain why it is acceptable to rewrite this equation as a square root, $y \approx 0.33\sqrt{x}$.

12. A power regression is only appropriate when there is no horizontal or vertical translation. How can you determine from the context of the pendulum problem that the parent function $f(x) = \sqrt{x}$ has not been translated?

LESSON 26-1 PRACTICE

13. Graph the function $f(x) = 2x^{\frac{1}{2}}$. Then use at least four points from your graph to sketch f^{-1} on the same coordinate grid.

14. Give the domain and range of the function and its inverse in Item 13. Write your answers using inequalities, set notation, and interval notation.

15. Find f^{-1} for the function in Item 13 algebraically. Give restrictions for the inverse if there are any and explain.

16. Consider the data in the table below.

x	2	5	7	10	12	15	18	20
y	4.2	6.7	7.9	9.5	10.4	11.6	12.7	13.4

a. Assuming a square root function in the form $f(x) = a\sqrt{x}$ is a good fit for the data, make a conjecture about the value of *a*. Explain your reasoning.

b. Use a graphing calculator to perform a square root regression. Write your answer using a rational exponent and then using a radical. Round all values to hundredths.

17. Model with mathematics. Determine whether a square root function is a good model for the data shown in the table below. Explain your reasoning. If a square root model is not appropriate, offer an alternative model and explain how you arrived at this model.

x	1	2	5	10	12	20	24	31	36
y	2	2.5	3.4	4.3	4.6	5.4	5.8	6.3	6.6

17. A cube root function is a more appropriate model since a power regression on the calculator results in an exponent of 0.335, which is very close to $\frac{1}{3}$.

ACTIVITY 26 Continued

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to regression equations.

Answers

11. 0.49 rounds to 0.50, which can be written as a rational exponent: $x^{0.49} \approx x^{0.5} = x^{\frac{1}{2}} = \sqrt{x}$.

12. The graph will start at (0, 0) since a length of 0 inches will produce a period of 0 seconds. This means there is no translation.

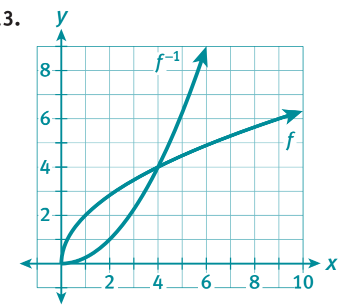
ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 26-1 PRACTICE

13.



14. domain: $x \geq 0$; $\{x \mid x \in \Re, x \geq 0\}$; $[0, \infty)$;
range: $y \geq 0$; $\{y \mid y \in \Re, y \geq 0\}$; $[0, \infty)$

15. $f^{-1} = \frac{x^2}{4}$ for $x \geq 0$ since the domain of the inverse should match the range of the original function.

16. a. The value of *a* must be greater than 1 since the *y*-values are all positive and the graph has been vertically stretched.

b. $y = 2.97x^{0.50}$; $y = 2.97\sqrt{x}$

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand how to find the inverse of a square root function, how to graph both the square root function and its inverse, how to find their domain and range, and how to perform a power regression on square root functions on their graphing calculators. Students should be able to recognize the parent function $f(x) = \sqrt{x}$. Additionally, students should be able to find the value of *a* in the function of $f(x) = a\sqrt{x}$ by performing a regression.